Influence of acoustic energy walk-off on acousto-optic diffraction characteristics

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**Abstract**

Influence of acoustic beam energy walk-off on characteristics of Bragg diffraction of light is studied theoretically and experimentally by the example of a paratellurite single crystal. Two cases of isotropic and anisotropic light scattering are examined. Angular and frequency characteristics of acousto-optic interaction are calculated in wide ranges of Bragg angles and ultrasound frequencies by means of modified Raman–Nath equations. It is shown that the walk-off can substantially change the width of angular and frequency ranges, resulting in their narrowing or broadening subject to position of the operating point in the Bragg angle frequency characteristic. Coefficients of broadening are introduced for characterization of this effect. It is established that frequency dependences of the broadening coefficients are similar to the Bragg angle frequency characteristics. Experimental verification of the calculations is carried out with a paratellurite cell of 10.5° crystal cut.

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1. Introduction

At the present time, acousto-optic (AO) methods of optical radiation control are widely applied in various areas of science and engineering. AO effect permits effective varying amplitude, frequency, phase and polarization of a light wave, as well as changing direction of its propagation. Such AO instruments as modulators, deflectors and filters stand out with high speed of operation, low driving power, construction simplicity and reliability. These advantages have provided their broad usage not only in laser physics, but also in controlling and measuring instrumentation, ecology, medicine, defence technology, etc. [1–4].

In modern acousto-optics, crystalline materials are used predominantly for manufacturing AO instruments. Among these materials, such crystals as paratellurite (TeO₂), tellurium (Te), calomel (Hg₂Cl₂), mercurous bromide (Hg₂Br₂), TAS (Tl₃AsSe₃) and some others occupy a prominent place, which are distinguished by exceptionally large anisotropy of acoustic properties [5–8]. The acoustic anisotropy influences sound propagation in different ways. First, it changes the structure of the acoustic field. For example, because of the anisotropy, energy divergence of the acoustic beam exceeds diffraction divergence by the factor 52 when the beam propagates in direction [110] of paratellurite. In this case, the far-filed radiation zone is situated inside the crystal cell. On the other hand, in the same crystal there are directions where diffraction divergence is compensated by the acoustic anisotropy almost completely [9–11]. Second, the acoustic anisotropy leads to the walk-off of acoustic energy [12]. For example, the walk-off angle may be as great as 74° in paratellurite and 49° in tellurium [7,8]. The acoustic walk-off does not change the AO phase matching condition. Maybe this is the reason that the acoustic walk-off effect was not taken into account in acousto-optics until recent time. The interest to this problem has appeared only in the last years [13–16].

It should be noticed that the problem of light diffraction by an acoustic walk-off beam is close to the problem of light diffraction on phase slanted gratings created by the holographic method in a thick photographic emulsion or in a photorefractive material [17–25]. This purely optical problem has been studied in detail by many researchers beginning from Kogelnik’s classical paper [17]. However, there are a number of important distinctions in these two problems. First, the ultrasonic phase grating is a dynamic structure, because all its parameters (amplitude, frequency or phase) may be varied electrically. Functioning of any AO device is based on this feature. Therefore the calculation of amplitude, angular, frequency and spectral characteristics is an obligatory procedure at designing AO devices [1–3]. Second, the optical
anisotropy of crystals essentially changes the AO phase matching condition. In the case of so called anisotropic diffraction, the dependence of the Bragg angle on ultrasound frequency turns out to be different depending on the operating point position. Analysis of light diffraction by optically anisotropic slanted gratings is fulfilled only in articles [20,21], however the obtained results are of little use for acousto-optics.

As regards publications [13–16] which have a direct relationship to AO problems, they are devoted to some particular questions and do not give a general insight into the influence of the acoustic walk-off on AO characteristics. Zakharov et al. [13,15] studied the transition from the Raman–Nath to Bragg diffraction regime with increasing the Klein–Cook parameter and showed that the angular selectivity of isotropic diffraction grows faster in the case of acoustic slanted beam. Besides, they revealed a peculiarity unknown before, which consists in appearing asymmetry of the diffraction pattern at the normal optical incidence on the acoustic beam. Kastelik et al. [16] analyzed operation of an anisotropic paratellurite deflector and established that both optical and acoustic anisotropy of crystals change symmetry of the deflector bandwidth to the magnitude of about 1% or less.

The given paper is a continuation of the research presented in our work [14]. In the first part of the paper, we demonstrate derivation of coupled-wave equations which take into account the acoustic walk-off. These modified Raman–Nath equations are valid for isotropic and anisotropic diffraction and allow calculating AO characteristics in the Raman–Nath, Bragg, and intermediate regimes of diffraction. On the basis of these equations, detailed computations are carried out of angular and frequency characteristics in a wide range of Bragg angles and ultrasound frequencies. To estimate the influence of the acoustic walk-off, we have introduced coefficients of broadening which show the extent of characteristic bandwidth alteration in comparison with the case of walk-off absence. Experimental verification of the calculations is carried out with 10.5° paratellurite cell.

2. Modified Raman–Nath equations

Fig. 1 illustrates the statement of the problem. We consider, as it is conventional for acousto-optics, 2-D variant of AO interaction, supposing that a monochromatic homogeneous acoustic field fills the space between two parallel planes \(x = 0\) and \(x = l\). The acoustic beam propagates along the \(z\) axis with the walk-off angle \(\alpha\), so that the wave vector \(K\) is directed at the angle \(z\) relative to the \(x\) axis. A plane light wave with vacuum wavelength \(\lambda\) falls on the acoustic column at the angle \(\theta_0\). The acoustic wave changes, in accordance with the photoelastic effect, the relative dielectric constant \(\varepsilon\) of the medium as

\[
e(\varepsilon, t) = n^2 + 2n\Delta n \sin (\Omega t - Kx - Kz + \Phi),
\]

where \(n\) is the static index of refraction, \(\Delta n\) is the amplitude of its variations caused by the ultrasonic wave, \(\Omega = Kv = 2\pi f\) is the acoustic frequency, \(V\) is the sound velocity, and \(\Phi\) is the initial acoustic phase. The wave equation for the electric field strength \(E\) in the medium perturbed by the acoustic wave can be written in the form

\[
\frac{\partial^2 E}{\partial x^2} - \frac{\partial^2 E}{\partial z^2} + \frac{1}{c^2} \frac{\partial^2 (\varepsilon E)}{\partial t^2},
\]

where \(c\) is the light velocity in vacuum.

In acousto-optics, the method of coupled waves is most extensively employed for analysis of AO effects [1–3]. In this method, the periodic variations of dielectric constant \(\varepsilon\) (1) are considered as a small perturbation which provides coupling between undisturbed eigenmodes of the medium. Therefore the solution of Eq. (2) is sought in the form of a sum of plane waves with frequencies \(\omega_p\) and wave vectors \(k_p\), and relative amplitudes \(C_p\):

\[
E = E_0 \sum_{p=-\infty}^{\infty} C_p \exp \left[ j(\omega_p t - k_p x - k_p z) \right].
\]

Substituting (3) into (2) yields:

\[
\sum_p \frac{d^2 C_p}{d x^2} - 2j kp \frac{d C_p}{d x} \exp \left[ j(\omega_p t - k_p x - k_p z) \right] = \frac{n\Delta n}{c^2} \sum_p \left\{ C_p (\omega_p + \Omega)^2 \exp \left[ j(\omega_p + \Omega) t - (k_p + K) x \right] - (k_p + K) x \left[ \right] \right\} - \frac{n\Delta n}{c^2} \sum_p C_p (\omega_p - \Omega)^2 \exp \left[ j(\omega_p - \Omega) t \right] - (k_p - K) x \left[ \right]\right\}. (4)
\]

In all cases of AO interaction the dielectric constant perturbations are so weak that the condition of slow changes of amplitudes \(C_p\) with the coordinate \(x\) is satisfied very well. Therefore, we may suppose that

\[
\left| \frac{d^2 C_p}{d x^2} \right| < \left| kp \frac{d C_p}{d x} \right|.
\]

This gives grounds to neglect the second derivatives in Eq. (4). The next step consists in taking into account orthogonality of harmonic functions in (4) [26]. As a result, we obtain the set of coupled equations

\[
\frac{d C_p}{d x} - \frac{qe_p}{2} \left\{ C_{p+1} \exp \left[ j(q \eta_p x - \Phi) \right] - C_{p-1} \exp \left[ -j(q \eta_p x - \Phi) \right] \right\},
\]

\[
(p = 0, \pm 1, \pm 2, \ldots)
\]

with additional conditions:

\[
\omega_p = \omega_0 + p\Omega, \quad \text{(Doppler effect)}
\]

\[
k_{p+1} z = k_p z + K_z \quad \text{or} \quad k_{p+1} \sin \theta_0 \sin \phi_p = k_p \sin \theta_0 + K \cos \alpha.
\]

where \(\theta_0\) are the angles defining directions of the diffracted waves (Fig. 1).

System (6) indicates that every diffraction maximum \(p\) interacts only with two adjacent ones \(p + 1\) and \(p - 1\). Two-phonon and higher transitions are disregarded in conventional acousto-optics [1–3]. The diffraction efficiency is determined by two sets of parameters: \(q_p\) and \(\eta_p\). Coefficients of AO coupling \(q_p\) are defined by the relationship:

\[
\text{Fig. 1. Geometry of light diffraction by acoustic beam in presence of walk-off effect.}
\]
\[ q_p = \frac{\alpha_0^2 n \Delta n}{k_p c^2} = \frac{2 \pi n \Delta n}{\lambda \cos \theta_p} = \frac{\pi}{\lambda \cos \theta_p} \sqrt{2M}, \]

where \( P_0 \) is the acoustic power density and \( M \) is the AO figure of merit. The figure of merit is the main characteristic of AO medium defining its suitability for AO applications. The mismatch parameters \( \eta \) characterize the degree of AO phase matching violation:

\[ \eta_p = k_p - k_{p+1,1} - k_p \cos \theta_p - K \sin \alpha - k_{p+1} \cos \theta_{p+1} \]

\[ = k_p \cos \theta_p - K \sin \alpha - \sqrt{k_{p+1}^2 - (k_p \sin \theta_p + K \cos \alpha)^2}. \]

(10)

The phase matching condition \( \eta_p = 0 \) determines the Bragg angle \( \theta_p \) for the \( +1 \)st order of diffraction:

\[ \sin (\theta_p - \alpha) = -\frac{jf}{2n_0 V} \left[ 1 + \frac{V^2}{\lambda^2 f^2} (n_0^2 - n_l^2) \right]. \]

(11)

For numerical calculations, it is convenient to introduce the following dimensionless values:

\[ X = \frac{X}{\lambda}, \quad \Gamma_p = \frac{2 \pi n \Delta n}{\lambda} \frac{l}{\cos \theta_p}, \]

\[ R_p = \eta_p l = \frac{2 \pi n}{\lambda} \left[ n_p \cos \theta_p - \frac{2f}{V} \sin \alpha - \sqrt{\frac{n_k^2 - n_p^2}{\lambda^2 - (2f V \sin \alpha)^2}} \right]. \]

(13)

Then system (6) takes the form:

\[ \frac{dG}{dX} = \frac{\Gamma_p}{2} \left\{ C_{p-1} \exp[j(R_p X - \Phi)] - C_{p-1} \exp[-j(R_p X - \Phi)] \right\}. \]

(14)

This system describes interaction of light and ultrasound in any diffraction regime (Raman–Nath, Bragg or intermediate) and is true for both isotropic and anisotropic diffraction. In the latter case, variations of the indices of refraction \( n_p \) with the angles \( \theta_p \) should be taken into consideration.

In acousto-optics, it is commonly accepted to measure incidence angles from the acoustic wave front [1]. Introducing just these angles \( \phi_p = \theta_p - \alpha \), we see that this change of variables does not influence the view of Eq. (14); however expressions (12) and (13) take a little different form:

\[ \Gamma_p = \frac{2 \pi n \Delta n}{\lambda} \frac{l}{\cos (\phi_p + \alpha)}, \]

\[ R_p = \frac{2 \pi l}{\lambda} \left[ n_p \cos (\theta_p + \alpha) - \frac{2f}{V} \sin \alpha \right] \left[ \frac{n_k^2 - n_p^2}{\lambda^2 - (2f V \sin \alpha)^2} \right]. \]

(16)

At the same time, relation (11) is transformed to the well-known expression that governs the Bragg angles in anisotropic media [1,27]. Consequently, one can conclude that the acoustic walk-off does not affect the phase matching condition, but it has to change angular and frequency characteristics through the values \( \Gamma_p \) and \( R_p \).

It should be noticed that the coupled wave Eq. (14) have a conventional form [1,26,28]. The difference consists in parameters \( \Gamma_p \) and \( R_p \) which depend on the walk-off angle \( \alpha \).

For the Bragg regime, when the incident light is scattered into +1st or –1st diffraction orders, Eq. (14) take the forms:

\[ \frac{dG}{dX} = -\frac{f}{2C_0} C_1 \exp[j(R_0 X - \Phi)] \]

\[ \frac{dG}{dX} = -\frac{f}{2C_0} C_0 \exp[-j(R_0 X - \Phi)], \]

\[ \frac{dG}{dX} = -\frac{f}{2C_0} C_0 \exp[-j(R_1 X - \Phi)]. \]

(17)

Solving these equations leads to the following expressions for the diffraction efficiency \( \zeta \):

\[ \zeta_{1,1,1} = C_{1,1,1}(l)C_{1,1,1}(l) = \frac{f^2}{4} \sin^2 \left( \frac{1}{2\pi} \sqrt{\frac{\Gamma_1}{\Gamma_1 + 1} + \frac{R_1}{R_1 + 1}} \right). \]

(18)

Thus, the diffraction efficiency is determined by parameters \( l \) and \( k \) which depend on the walk-off angle \( \alpha \). However, the dependence \( \Gamma(z) \) is trivial; the AO effect is proportional to the path length of optical rays in the acoustic field \( L_{ao} = \int \cos (\phi_0 + \alpha) \) at light scattering into the +1st order and \( L_{ao} = \int \cos (\phi_0 + \alpha) \) at scattering into the –1st order. It should be taken into consideration that the full diffraction angle between zero and ±1st orders is usually smaller than 10°, so that with good precision we can suppose \( \Gamma_0 \approx \Gamma_0 \approx \Gamma_{1,1,1} \).

Eq. (18) coupled with (15) and (16) permits calculating angular, frequency and spectral characteristics of Bragg AO interaction for variants of both isotropic and anisotropic diffraction.

Expressions (8) and (10) for \( p = 0 \), 1 can be treated as \( x-\alpha \) projection of the vector relationship:

\[ k_1 = k_0 + K + \eta_0, \]

(19)

where \( \eta_0 \) is the mismatch vector oriented orthogonally to the vertical boundaries of the acoustic beam. The length of this vector determines the +1st order diffraction efficiency; the longer this vector, the lower the scattered light intensity. The region of the AO interaction is usually defined by the condition \( |\eta_0| = |R_0| \leq \pi \).

3. Isotropic diffraction

The influence of the acoustic walk-off on angular characteristics at AO isotropic diffraction was examined by Zakharov et al. [13,15]. The calculations were carried out for a slow acoustic mode in a paratellurite single crystal, propagating at an angle of 7.5° to the [110] direction in the (001) crystallographic plane. This acoustic mode indicates a uniquely great magnitude of the walk-off angle: \( \alpha = 74° \). Since the analysis was fulfilled only for this crystal cut, the results obtained do not give a comprehensive idea of walk-off impact on AO characteristics. In this section we demonstrate a general solution of the problem.

For the case of isotropic Bragg diffraction the phase mismatch \( \eta_0 \) is determined by the following equation:

\[ \eta_0^2 - 2 \eta_0 k \cos (\phi_0 + \alpha) - K (k + 2k \sin \phi_0) = 0, \]

(20)

which directly results from Eq. (16) subject to \( k_0 = k_1 \equiv k = 2 \eta_0 \). This equation can be simplified with taking into account the condition \( K \ll k \) which is true up to about 1 GHz. As a result we have:

\[ \phi_0 = \frac{1}{2K} \left( -\eta_0^2 + 2 \eta_0 k \cos \alpha - K^2 \right). \]

(21)

Hence, fixing the acoustic frequency \( f \) and, consequently, the Bragg angle \( \phi_0 = -K + k/2 - \iota f/2V \), we obtain the angular bandwidth as

\[ \Delta \phi = 2 \eta_0 \max \cos \alpha / K = (A/l) \cos \alpha, \]

(22)

where \( \eta_0 \max = \pi/l \) and \( A \) is the acoustic wavelength. For characterizing the walk-off influence, let us introduce a coefficient \( B_\alpha = \Delta \phi / \phi_0 \), which \( \Delta \phi \) is the angular bandwidth in the real situation and \( \phi_0 \) is the bandwidth at \( \alpha = 0 \). It follows from (22) that \( B_\alpha = \cos \alpha \). The result obtained permits a simple physical explanation. According to Gordon’s interpretation of AO interaction in isotropic media [29], the AO angular range is defined by the acoustic beam divergence \( \phi_0 = A/l \), where \( l \) is the acoustic beam width and, simultaneously, the piezoelectric transducer width. In the acoustically anisotropic medium, the situation is the same: \( \phi_0 = \phi_0 \), but \( \phi_0 = A/l_{ac} = (A/l) \cos \alpha \) because the angular spectrum of the acoustic beam is determined by the transducer width \( L_{ac} = l/ \cos \alpha \) independently of the walk-off angle \( \alpha \) [11].
The AO frequency range $\Delta f$ can be found from (21) as well. For this, the incidence angle should be fixed as $\theta = \theta_0 = -K_0/2k$. Supposing high frequency selectivity ($\Delta f \ll f_0$), we yield

$$K = K_0 + \frac{1}{K_0} (-\eta_0' + 2\eta_0 k \cos \alpha).$$  

(23)

Hence it follows that

$$\Delta f = \frac{2nV}{2l_0} \cos \alpha, \quad B_f = \frac{\Delta f(\alpha)}{\Delta f(\alpha = 0)} = \cos \alpha.$$  

(24)

Consequently, in the case of isotropic diffraction the acoustic walk-off results in narrowing both angular and frequency ranges of AO interaction by the factor $1/B_f = 1/B_f = 1/\cos \alpha$ independently of acoustic frequency and optical wavelength. For example, if $\alpha = 70^\circ$ the narrowing amounts to 2.9 times. Thus, we see that the acoustic walk-off can give a very significant effect. Nevertheless, the calculation of AO characteristics can be made with using conventional formulas only if the transducer width $L_t = l/\cos \alpha$ will be employed instead of the acoustic beam width $l$.

4. Anisotropic diffraction

Anisotropic diffraction of light in crystals gives quite other regularities. Because of difficulties of analytical analysis, we present here results of computer calculations fulfilled by the example of paratellurite crystal.

Presently, paratellurite is a basic material for fabrication of AO devices intended for operation in visible and IR spectral ranges (the transparent region is $0.35 - 5 \mu m$ [13]). The crystal is distinguished by an exceptionally high value of AO figure of merit $M = 1200 \cdot 10^{-18} \text{s/g}$, which is attained at anisotropic diffraction by a shear acoustic mode propagating along the [110] direction. However, due to strong acoustic anisotropy, the acoustic field in this case appears to be very non-homogeneous [11]. For eliminating this drawback, skew crystal cuts are usually applied in which the vector $K$ is rotated through the angle $\chi$ to the crystallographic axis $Z$ in the plane (110). In this case, the acoustic beam proves to be sufficiently homogeneous, but the energy walk-off appears. The walk-off angle attains its maximum value $\beta_{max} = 57.3^\circ$ at $\chi = 17^\circ$. In our experiments an AO cell was employed with the cut angle $\chi = 10.5^\circ$ and the walk-off angle $\alpha = 54.6^\circ$. Calculations were carried out also for the cuts with $\chi = 5^\circ$ ($\alpha = 40.5^\circ$) and $\chi = 15^\circ$ ($\alpha = 57.2^\circ$).

Fig. 2a demonstrates the Bragg angle $\phi_B$ as a function of the acoustic frequency $f$ for the case $\chi = 10.5^\circ$ and $\lambda = 0.6328 \mu m$. The four branches correspond to different polarizations of incident light ($o$ and $e$) and its scattering into different orders of diffraction (+1st or -1st ones). For example, branch +1e conforms to anisotropic diffraction of the optical wave with extraordinary polarization into the +1st order. Points T and D show areas of tangential ($d\phi_B/df \rightarrow \infty$) and deflector ($d\phi_B/df \rightarrow 0$) geometries of AO interaction, which are used in wide-angle videofilters and deflectors, whereas points M at the zero Bragg angle indicate optimal areas for modulators [1]. The calculation is carried out with having regard to the optical activity effect in the TeO$_2$ crystal [26]. The crosses point out results of experimental measurements. Our experimental AO cell was optimized for operation as a videofilter, so its input facet was oriented orthogonally to the optical beam falling at the angle $\phi_B = -14.5^\circ$. The cell geometry made it possible to measure angular and frequency characteristics in the range of Bragg angles from $-20^\circ$ to $+5^\circ$ and acoustic frequencies from 50 to 150 MHz.

Fig. 2b presents analogous calculations for the case $\chi = 15^\circ$. The tendency in changing the curves is quite clear: with increasing the cut angle $\chi$, the specific points T, D and M shift to the area of higher magnitudes of frequencies and Bragg angles.

4.1. Frequency and angular characteristics

The frequency characteristic of AO interaction or, in other words, the dependence of the diffraction efficiency $\zeta$ on the ultrasonic frequency $f$ is one of principal characteristics of AO devices. This dependence defines the operating frequency range and, thereby, the operation speed.

Fig. 3 demonstrates the frequency bandwidth as a function of the Bragg angle $\phi_B$ for the variant $\chi = 10.5^\circ$. The acoustic beam width was assumed to be $l = 0.7 \text{ cm}$ in accordance with the transducer width $L_t = l/\cos \alpha = 1.2 \text{ cm}$ in our experimental cell. Solid curves 1 show the bandwidth $\Delta f_0$ in the real situation with the walk-off angle $\alpha = 54.6^\circ$. Using the curves in Fig. 2a these graphs can be recalculated in dependences $\Delta f_0$ on the central frequency $f_0$ of the frequency band.

Fig. 3a relates to the +1e branch in Fig. 2a. It is seen that the bandwidth $\Delta f_0$ rises sharply with approaching the deflector geometry and reaches a magnitude of 16.6 MHz at point D where $\phi_B = -5.65^\circ$ and $f_0 = 253.5 \text{ MHz}$. For comparison, dashed line 2 shows the analogous dependence $\Delta f_0(\phi_B)$ for the case when the walk-off is absent. Thus, we see that the acoustic walk-off decreases the frequency range of AO interaction in the area of the Bragg
angles from $-5.65^\circ$ to $-30.1^\circ$, but at $\varphi_B < -30.1^\circ$ the situation becomes inverse.

Similar regularities are observed for the $-1\alpha$ branch. In the range $\varphi_B > -28.2^\circ$ the walk-off causes narrowing the frequency band, while at $\varphi_B < -28.2^\circ$ the bandwidth $\Delta f_u$ becomes broader than $\Delta f_0$. Maximum values of the band are attained at $\varphi_B = 0$ because the characteristic $\varphi_{gf}$ has here a minimal steepness. However, since the frequency band is of an order narrower than at the $+1\alpha$ branch, the $-1\alpha$ branch is not employed in AO deflectors.

Experimental studies were carried out only at those Bragg angles which were allowed by AO cell geometry. Experimental results presented in Fig. 3a and b indicate a good agreement with the numerical calculation.

The regularities revealed are seen in Fig. 3c and d as well, where frequency bandwidth calculations are depicted for the $-1\alpha$ and $+1\alpha$ branches respectively. A strong widening of the frequency band up to $\Delta f_u = 20.4$ MHz takes place in the region of deflector geometry at the frequency $f_0 = 126.7$ MHz. However, contrary to the cases $+1\alpha$ and $-1\alpha$ here the band $\Delta f_u$ is narrower than $\Delta f_0$ in all calculated area. Experimental verification of these curves failed owing to irregularity of the transducer frequency characteristic.

Yet another important characteristic of AO interaction is the dependence of the diffraction efficiency $\zeta$ on the incidence angle $\varphi_0$, which defines the angular range of interaction $\Delta \varphi$. The angular ranges $\Delta \varphi_{+1\alpha}$ (curves 1) and $\Delta \varphi_{-1\alpha}$ (curves 2) calculated for all branches of AO interaction are presented in Fig. 4. For these plots, points T of tangential geometry are peculiar because they correspond to extremely low angular selectivity. For example, in the case of the $+1\alpha$ branch (Fig. 4a) the angular range attains the magnitude $\Delta \varphi_{+1\alpha} = 1.9^\circ$. For the branches $-1\alpha$ and $+1\alpha$ (Fig. 4c and d) the ranges are even greater due to lower acoustic frequency.

Experimental results were obtained only for the upper branches $-1\alpha$ and $+1\alpha$ in the area of small Bragg angles. The lower branches give excessively wide angular characteristics. Moreover, near the tangential geometry they take a very complicated two-humped structure.

Much like the case of the frequency characteristics, curves $+1\alpha$ and $-1\alpha$ have points of intersection where the ranges $\Delta \varphi_{+1\alpha}$ and $\Delta \varphi_{-1\alpha}$ prove to be equal. These points conform to the Bragg angle $\varphi_B \approx -30^\circ$ as well. The peculiarity revealed becomes realizable from the vector diagrams in Fig. 5 constructed according to Eq. (19). Here the vector $\vec{n}_\varphi$ represents the mismatch that would be, if the walk-off is absent. The direction of this vector is orthogonal to the wave vector $\vec{K}$. The mismatch vector $\vec{n}_\varphi$ depicts the real situation with the walk-off angle $\varphi$. This vector is perpendicular to the boundaries of the acoustic beam and forms the angle $\varphi$ with the vector $\vec{n}_\varphi$. At the angle $\varphi_B \approx -30^\circ$ these two vectors have the same length and, in consequence of this feature, the angular ($\Delta \varphi_{+1\alpha}$ and $\Delta \varphi_{-1\alpha}$) and frequency ($\Delta f_{+1\alpha}$ and $\Delta f_{-1\alpha}$) ranges have equal values. It should be noticed that the points, where the acoustic walk-off does not influence AO characteristics, are determined by the crystal cut and, in more general sense, by acoustic and optical anisotropy of a concrete crystal. In paratellurite, this effect occurs at the Bragg angle $\varphi_B \approx -22^\circ$ for the cut angle $\chi = 5^\circ$ and $\varphi_B \approx -32^\circ$ in the case of $\chi = 15^\circ$.

4.2. Broadening coefficients

Since the angular and frequency ranges depend on many parameters which characterize both AO crystal itself and AO cell made from this crystal, more information is given by broadening
coefficients $B_u$ and $B_f$ which characterize the influence of the acoustic walk-off on AO characteristics. These coefficients can take values more or less than unity depending on crystal cut, incidence angle and acoustic frequency.

Fig. 6 displays the coefficients $B_u$ as functions of the AO matching frequency $f_0$, i.e. the frequency for which the chosen incidence angle is the Bragg angle: $\phi_0 = \phi_B$. The calculations are carried out for two variants of crystal cuts $\chi = 10.5^\circ$ (a) and $\chi = 15^\circ$ (b) in the range of the Bragg angles from $-60^\circ$ to $+60^\circ$. The ordinate scale is chosen in the reverse direction. Comparing this figure with Fig. 2, one can note their astonishing similarity despite the fact that the value $B_f$, which is plotted along the vertical axis, characterizes acoustic anisotropy influence and has no relevance to the Bragg angles. One can suppose that this similarity is caused indirectly by optical anisotropy (shape of the refractive indices surface) which determines condition (19). At the same time, we see some distinctions. In the tangential regions, the curves for $o$ and $e$ polarizations do not cross; they only touch each other. Besides, they have inverted location in the region of small Bragg angles in comparison with Fig. 2.

Another interesting and unexpected peculiarity was revealed at calculation of angular characteristics. It has been established that the dependences $B_u(f_0)$ coincide with $B_f(f_0)$ not only qualitatively, but quantitatively as well (within limits of computation accuracy). This is astonishing because the angular and frequency characteristics have substantial differences, as seen from Figs. 3 and 4.

An important deduction follows from Fig. 6: the influence of the acoustic walk-off at anisotropic diffraction is not vanishingly small; in the region calculated the coefficients $B$ change from 0.068 (15 times narrowing) to 2 (2 times broadening). This effect is caused by joined action of two reasons: (1) change of the mismatch vector $\eta_0$ direction due to the walk-off and (2) change of the optical path length in the slanted acoustic beam.
and are introduced which describe the change in the width of frequency and angular ranges respectively.

In the case of isotropic diffraction, the dependence of these coefficients on the walk-off angle $\alpha$ is described by simple relationship $B_l = B_0 = \cos \alpha$ independently of acoustic frequency and optical wavelength. This means that the calculation of AO characteristics can be made with using conventional formulas with replacing the acoustic beam width $l$ by the transducer width $L_T = l/\cos \alpha$.

Anisotropic diffraction gives much more complicated dependences $B(\alpha)$. Subject to the crystal cut, acoustic frequency and incident light polarization the coefficients $B$ can take values more or less than unity, demonstrating thereby effects of broadening or narrowing the AO interaction ranges. Numerical calculations carried out for paratellurite crystal have shown that the influence of the acoustic walk-off at anisotropic diffraction is not vanishingly small; this effect can change the AO interaction range several times. Thus, our research has shown that the acoustic walk-off should be taken into consideration when designing AO devices.

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