Theory of acousto-optical Bragg diffraction of ultra-short laser pulses in dispersive crystals

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A theoretical analysis of collinear acousto-optical diffraction of ultra-short laser pulses for strong interaction is presented. The model simultaneously considers Bragg diffraction and group delay of wave packets. The new theoretical approach reveals transformation of the pulse envelope and pulse spreading even in a medium without group delay dispersion. These results are crucial for precise characterisation of acousto-optical dispersive delay lines.

Introduction: Acousto-optical (AO) devices are widely used in optical communications [1], spectral image processing [2], femtosecond laser pulse controlling [3] etc. Conventionally, Bragg diffraction is described by the coupled-wave equations [4]. During AO scattering of ultra-short laser pulses in a strong field limit, spatio-temporal invariance of energy transfer between coupled waves can be inferred. That phenomenon originates from the group delay between the incoming and diffracted ultra-short pulses. Similar space–time coupling violation in a strong pulsed acoustic field was first observed in AO modulators [5], then studied theoretically [6].

The problem of group delay influence on wave coupling was studied in nonlinear optics [7], however, it is novel for acousto-optics. A comprehensive theory of Bragg AO diffraction in crystals with strong material dispersion is necessary for the precise characterisation and accurate design of AO dispersive delay lines for femtosecond laser systems [3, 8–10]. The existing theory of strong AO interaction [4] assumes equal conditions of direct and backward phase matching, hence it is not suitable for ultra-short pulses. We propose a combined wave theory for Bragg diffraction of Fourier transform limited ultra-short optical pulses to solve that problem.

Theory: Consider an optical wave packet travelling in a crystal collinearly to the acoustical wave along the z-direction. The electrical field vector in the interaction medium is considered as a sum of two eigenwaves \( \mathbf{E}_p \) with slowly varying complex amplitudes \( \mathcal{A}_p(t; z) \) [11, 12]:

\[
\mathbf{E}(t; z) = \sum_p \mathcal{E}_p \mathcal{A}_p(t; z) \exp[i(wt - kz)]
\]

where the subscript \( p = 0 \) corresponds to the incident wave (0th order) and \( p = 1 \) corresponds to the diffracted wave (1st order); wave numbers \( k_p \) are coupled with the frequency of light via the dispersion equation \( k_p = n_c p_0 / c \), where \( c \) is the speed of light in vacuum and the \( n_c \) are the refractive indices of the crystal. In this Letter, the analysis concerns only the first order of the dispersion approximation, and Bragg diffraction of light by ultrasound is assumed. Group velocities of the eigenwaves in the crystal are equal to \( u_p = (\partial k_p / \partial w)^{-1} \); group delay between the fast and the slow mode in the crystal is characterised by the parameter \( u = u_0 / (u_0 - u_1) \). For definiteness we take \( u_0 > u_1 \) that corresponds to the incident fast eigenwave and the diffracted slow eigenwave.

According to Maxwell’s theory, the optical field obeys the wave equation

\[
\frac{\partial^2 \mathbf{E}}{\partial t^2} - c^2 \nabla^2 \mathbf{E} = 0
\]

where the permittivity tensor \( \mathbf{E}(t; z) = E_p(t; z) \Delta \epsilon \mathbf{E}(t; z) \) corresponds to a homogeneous transparent medium with an acoustically induced phase grating. Substitution of (1) into (2) provides a general equation for the envelopes of a wave packet, \( \mathcal{A}_p(t; z) \), in a crystal with periodical perturbation of refractive indices. Under the assumptions made, the following equations may be derived in the normalised frame of reference \( \mathcal{E}_p = (-1)^p \mathcal{E}_0 / L \), \( \eta_p = (\tau - z) / \eta_0 \):

\[
\frac{\partial^2 \mathcal{A}_p}{\partial \eta_p^2} + U \frac{\partial \mathcal{A}_p}{\partial \eta_0} + \frac{\eta_0^2}{4} \frac{\partial^2 \mathcal{A}_p}{\partial \eta_0^2} = 0
\]

where \( \mathcal{Q} = 0 \), \( \mathcal{U} = \frac{\partial \mathcal{A}_p}{\partial \eta_0} \) and \( \eta_0 \) and \( \mathcal{U} \) are the characteristic parameter and drive, respectively. Equation (3) was derived for exact phase matching [11]. These equations provide a generalised representation of coupled wave equations and the equation for the envelope of the wave packet with the approximation of first-order dispersion. The initial conditions are given at the input of the interaction region, \( \eta_p = 0 \), and the upper limit of the spatial domain is \( \eta_p = 1 \).

To solve the problem, it is necessary to specify initial conditions for the 0th diffraction order:

\[
A_0(\eta_0; \zeta_0 = 0) = A_0(\eta_0), \quad \partial_0 A_0(\eta_0; \zeta_0 = 0) / \partial \zeta_0 = 0 \quad (4)
\]

and for the first diffraction order:

\[
A_1(\eta_1; \zeta_1 = 0) = 0, \quad \partial_1 A_1(\eta_1; \zeta_1 = 0) / \partial \zeta_1 = -(\pi/2) Q A_0(\eta_0)
\]

where \( \eta_0 \) is the duration parameter of the initial pulse at \( \zeta_0 = 0 \).

Simulation and analysis: We search the solution of (3) for time-limited wave packets so the input pulse is localised around \( \eta_0 = 0 \). The typical group delay between the incident and the diffracted pulse is determined by the birefringence of the acousto-optical medium and equals \( L / u \) at the end of the interaction region, \( \zeta_1 = 1 \). The time domain must be taken wide enough to provide null boundary conditions for \( A_p(\eta_0; \zeta_1) \) and for its derivative \( \partial_0 A_p(\eta_0; \zeta_1) / \partial \zeta_1 \). Compared to second-order ordinary differential equations that are derived from coupled wave equations for stationary electromagnetic waves [4, 11], (3) is a partial differential equation, therefore its solution is a time-variable envelope of the wave packet. According to the definitions above, \( U = 1 \) if the fast eigenwave is incident as assumed. Conventional coupled wave equations can be represented as a particular case of (3) with \( U = 0 \). The solution is a wave packet with a constant shape of the envelope and a variable magnitude as a function of \( \zeta_1 \). It is known that optimal mode coupling and total energy transfer from mode 0 to mode 1 is observed at \( \zeta_1 = 1 \) if the coupling coefficient \( Q = 1 \).

The finite-difference method was used for solving (3). The solution with initial conditions (4) and (5) and initial pulse duration \( \eta_0 = 0.2 \) is plotted in Figs. 1a and b as a function of two co-ordinates, \( \zeta_1 \) and \( \eta_0 \). This choice of incident pulse width provides good accuracy null boundary conditions at the borders of the time domain, \( \eta_0 = 1.5 \), and well discriminated delay between the slow and the fast eigenwaves at \( \zeta_1 = 1 \). For analysis, generalised statistical pulse characterisation of the pulse duration was used [12].

The pulse width is shown in Fig. 1 by the solid lines. A sufficient spreading of pulses in both 0th and first diffraction orders is observed. This increase in pulse width is due to a group delay between fast and slow optical waves in the crystal. In our case, the group delay violates the spatio-temporal invariance of acousto-optical interaction, hence the front edge of the slow pulse interacts with the back slope of the fast pulse. As a result, efficient propagation velocity of the diffracted pulse is higher than \( u_1 \). Another effect is that the tail of the incident pulse is diffracted with a higher efficiency than its front, which causes transformation of the pulse envelope.

Diffraction efficiency is characterised in Fig. 2. Both peak power and pulse energy for the diffracted wave packet are sufficiently lower compared to the solution of the coupled wave equations for continuous waves. Moreover, both these functions have maxima at \( \zeta_1 < 1 \). The peculiarity of that diffraction regime is that the maximum of energy transfer from first order to 0th order is shifted in time relative to the maximum of the incident pulse. Profiles of the pulse envelopes are shown in Fig. 3; for each mode, intensity is plotted against normalised time \( \eta_0 \) in a co-moving frame of reference.
The output pulse at $|z_p| = 1$ in the first order is a flat-top wave packet, while the pulse in the 0th diffraction order has two separated peaks. This modelling shows that, to obtain the best pulse parameters at the output of the interaction region, the length of interaction must be reduced.

Conclusions: We have shown that introducing the group delay between the eigenwaves in the crystal dramatically changes the solution of coupled wave equations of Bragg diffraction for wave packets. The pulse width increases both for diffracted and non-diffracted beams, and coupling efficiency is lower than predicted for a stationary electromagnetic field. The described phenomena are not predicted either by theory of Bragg diffraction or by dispersion theory if applied separately. These results are especially important in engineering of AO dispersive delay lines for ultrafast optics and estimation of pulse shape distortions.

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