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Coupled-wave equations of Bragg diffraction for wave packets in dispersive media

Konstantin B. Yushkov* and Vladimir Ya. Molchanov*

*National University of Science and Technology "MISIS", Acousto-Optical Research Center, RUSSIA, 119049 Moscow, Leninsky prospect 4
†konstantin.yushkov@misis.ru

Abstract. We propose a unified theory of acousto-optical diffraction in crystals and wave packet propagation in dispersive media. This formal approach is crucial for precise characterization of light dispersive delay lines performance for femtosecond laser pulses. We found that dispersive phenomena in crystals influence acousto-optical coupling of modes. Modified coupled-wave equations were developed for description of acousto-optical interaction of ultrashort laser pulses. Analysis showed that group delay between the incident and the diffracted pulse results in considerable transformation of pulse envelope and decrease of diffraction efficiency. On the contrary, the influence of phase mismatch on the diffraction efficiency is lower than it is predicted by conventional theory of Bragg diffraction. As a result, the transmission bandwidth for ultrashort wave packets increases several times compared to that for stationary waves and the side lobes of the transmission function disappear.

Keywords: Femtosecond laser pulses, collinear diffraction, dispersion
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INTRODUCTION

Bragg diffraction of light by ultrasound was comprehensively examined for stationary and quasi-stationary electromagnetic fields [1, 2, 3]. Nevertheless, recent experimental progress in acousto-optical controlling of femtosecond pulse parameters established a demand in novel approach to the coupling-of-waves model of Bragg diffraction [4, 5]. The crucial peculiarity of ultrashort optical pulses is the susceptibility to different kinds of dispersive phenomena [6, 7]. For precise characterization of acousto-optical diffraction of ultrafast optical pulses it is necessary to consider coupling of optical modes simultaneously with the material dispersion of interaction media. In this work we demonstrate that the group delay between the incident and the diffracted pulse is an important factor that alters Bragg phase matching conditions. The influence of material dispersion on the conditions of Bragg diffraction was not studied before. Moreover, conventional theory of acousto-optical compression of ultrashort pulses is based on the approximation of weak collinear diffraction [8, 9, 10].

We will show that combination of coupled-wave equations of Bragg diffraction with the equation of wave packet propagation in dispersive media provides a new partial differential equation (PDE) problem that we call "Modified coupled-wave equations".

CONVENTIONAL THEORY

In is known that Bragg diffraction of plain optical waves is described by the coupled-wave ordinary differential equation (ODE). The amplitudes of the incident and the diffracted wave, \( A_0(z) \) and \( A_1(z) \), follow the equation [1, 2]

$$\begin{align*}
\frac{\partial A_0}{\partial z} &= \frac{q_0}{2} \exp(-i\Delta k z) \exp(-i\Phi) A_1; \\
\frac{\partial A_1}{\partial z} &= -\frac{q_1}{2} \exp(i\Delta k z) \exp(i\Phi) A_0,
\end{align*}$$

(1)

where \( q \) are the coupling coefficients, \( \Delta k \) is the phase mismatch, and \( \Phi \) is the acoustical phase. The equations are solved in the finite interval \( z \in [0, L] \). The solution of this problem satisfactorily describes most cases of interaction between continuous waves (CW). However, the output radiation of mode-locked (ML) lasers is a train of ultrashort optical pulses. In ultrafast optics, a method of slowly varying amplitude is used to describe propagation of a wave
packets in dispersive media [6]. In the second-order approximation of dispersion, the PDE for the envelope of the wave packet is the following:

\[ \left( \frac{\partial}{\partial z} + \frac{1}{u} \frac{\partial}{\partial t} \right) A - i\gamma \frac{\partial^2 A}{\partial t^2} = 0, \]

where \( u \) is the group velocity and \( \gamma \) is the group delay dispersion (GDD) [2]. The principal distinction between Eqs. (1) and (2) is the temporal dependence of the amplitude in the second case.

The existing theory of collinear acousto-optical ultrashort-pulse compression [8, 9, 10] uses two different approximations. On the one hand, the low-field approximation is used. In this case, the transmission function of the acousto-optical unit is proportional to the Fourier transform of the temporal profile of the acoustic wave in the interaction medium [8, 9, 11]. On the other hand, only the group delay due to acousto-optical diffraction is considered, however the problem of Bragg coupling is not solved for this approach [10]. In the next section, we will show that simultaneous consideration of Bragg coupling and dispersion is crucial for correct description of acousto-optical interaction.

### MODIFIED COUPLED-WAVE EQUATIONS

Equations (1) and (2) are both derived from the Maxwell equations, given correspondent permittivity of the medium \( \varepsilon(t, z) [2] \). For the case of acousto-optical interaction, the permittivity is considered as a perturbation of the stationary permittivity tensor by the acoustical wave due to the photoelasticity. Periodic spatial modulation of refractive indices causes mode coupling, while the temporal modulation is responsible for the frequency shift of the diffracted wave. On the contrary, in the theory of wave packet propagation, the permittivity of the medium is treated as a wavelength-dependent function. As well as we neglect nonlinear phenomena, it is possible to represent overall permittivity as a sum of frequency-dependent permittivity tensor \( \varepsilon(\omega) \) and an oscillating perturbation induced by ultrasound \( \Delta \varepsilon(t, z) \):

\[ \varepsilon(\omega, t, z) = \varepsilon(\omega) + \Delta \varepsilon(t, z). \]

After the substitution into the wave equation, the permittivity tensor given by Eq. (3) provides a general set of equations for the complex amplitudes of coupled electromagnetic waves:

\[
\begin{align*}
\left( \frac{\partial}{\partial z} + \frac{1}{u_0} \frac{\partial}{\partial t} \right) A_0 - i\gamma_0 \frac{\partial^2 A_0}{\partial t^2} &= \frac{q_0}{2} \exp(-i\Delta k z) \exp(-i\Phi) A_1; \\
\left( \frac{\partial}{\partial z} + \frac{1}{u_1} \frac{\partial}{\partial t} \right) A_1 - i\gamma_1 \frac{\partial^2 A_1}{\partial t^2} &= -\frac{q_1}{2} \exp(i\Delta k z) \exp(i\Phi) A_0.
\end{align*}
\]

Here, the dispersive coefficients \( u^{-1} \) and \( \gamma \) originate from the \( \varepsilon(\omega) \), while the coupling coefficients \( q \) are proportional to the magnitude of \( \Delta \varepsilon \).

One can see that Eq. (4) can be reduced to either Eq. (1) or Eq. (2) as particular cases. At a zero ultrasonic amplitude, \( q_0 = q_1 = 0 \), independent propagation of two eigenwaves in the crystal follows Eq. (2). On the contrary, for a single-frequency optical spectrum, \( \varepsilon(\omega) = \varepsilon(\omega_0) \), the temporal derivatives \( \partial A / \partial t \) and \( \partial^2 A / \partial t^2 \) tend to zero, and coupling of modes is described by conventional ODEs (1).

It is evident, that a generalization of Eq. (4) may contain high-order dispersion terms. Nevertheless, in this paper we do not consider the effect of high order dispersion on the Bragg coupling of modes. Moreover, the following analysis is restricted to the group delay only corresponding to the first-order dispersion, because even with this approximation, the influence of dispersion on the acousto-optical diffraction is dramatic.

### SIMULATION AND ANALYSIS

For the simulation, we used the approximation of first-order dispersion, i.e. neglecting inherent to the crystal GDD terms in Eq. (4). The set of two PDEs was transformed into two independent second-order PDEs with different initial conditions. In the normalized frame of reference \( \xi = \pm z/L, \eta = (t - z/u) |v|/L \), where \( v = u_0 u_1 / (u_0 - u_1) \) is the group delay, the following equation emerges:

\[ \frac{\partial^2 A}{\partial \xi^2} \pm U \frac{\partial^2 A}{\partial \eta \partial \xi} \pm i\pi H \frac{\partial A}{\partial \xi} + \frac{\pi^2}{4} \partial^2 A = 0, \]

\( U, H \) are parameters specific to the crystal and \( \pi \) is the spatial frequency of the acoustic wave. The parameters \( \xi, \eta, \) and \( \xi \) correspond to the spatial and temporal coordinates in the interaction with acoustic waves, respectively.

The parameters \( U, H \) are calculated from the known values of the acoustic wave frequency and spatial frequency, respectively. The solution of this PDE represents the propagation of electromagnetic waves in the interaction medium, taking into account the influence of dispersion on the acousto-optical process. The solutions are obtained using numerical methods, such as finite difference or finite element methods, and are compared with experimental results to validate the theoretical model.

In conclusion, the consideration of Bragg coupling and dispersion is crucial for the correct description of acousto-optical interaction. The simulation and analysis of the proposed theoretical model provide a deeper understanding of the interaction mechanisms and allow for the optimization of acousto-optical devices for various applications.
where plus sign is for envelope of the zeroth diffraction order, $A_0$, and minus sign is for envelope of the first diffraction order, $A_1$. Here the normalized coefficients are $U = |v|/\nu$, $H = \Delta k L/\pi$, and $Q = \sqrt{Q_0 q}(|L|/\pi)$. Dirichlet initial conditions were used for the incident wave, $A_0|_{\xi_1=0} = A_{in}(\eta_0)$, and Neumann initial conditions were used for the diffracted wave, $A_1|_{\xi_1=0} = 0$ and $\partial A_1/\partial \xi_1|_{\xi_1=0} = A_{in}(\eta_1)$. The second term in Eq. (5) makes the difference from the conventional ODE that is used for analysis of Bragg diffraction in CW regime. The initial conditions were taken for a sech$^2$ pulse shape, i.e. $A_{in}(\eta) = \text{sech}(\eta/\tau)$, where the FWHM pulse duration is 1.76$\tau$.

Equations (5) were solved numerically using a finite-difference method. In Fig. 1, the simulation results are presented for the case of phase-matched diffraction ($H = 0$). The coupling parameter $Q = 1$ corresponds to the first Bragg maximum in CW regime. One can see, that a sufficient transformation of pulse envelope is observed for both diffraction orders. Unlike the solution of conventional equations of Bragg diffraction, full coupling of modes does not take place for ultrashort wave packets. Moreover, maximum energy of the diffracted pulse are obtained at $z < 1$.

Numerical analysis of the presented data shows that maximum 60% of pulse energy can be diffracted in this case. Correspondent maximum would be observed at $z = 1$, if the coupling coefficient was equal to $Q = 0.85$.

Further analysis concerns the influence of phase mismatch on the diffraction parameters. The solution of Eq. (1) results in the following transmission function, $T = |A_1(L)/A_0(\xi)|^2$, according to Ref. [1]:

$$T(Q,H) = \frac{Q^2}{Q^2 + H^2} \sin^2 \frac{\pi}{2} \sqrt{Q^2 + H^2}. \quad (6)$$

The magnitude of diffraction efficiency is treated as a function of problem parameters, $Q$ and $H$. At a given magnitude of $Q$, the transmission function $T(H)$ characterizes the influence of phase mismatch on the diffraction efficiency. For wave packets, we defined the transmission function as a ratio of diffracted pulse energy to the incident pulse energy.

The transmission function calculated from the solution of Eq. (5) was compared to the theoretical solution Eq. (6) for high efficiency CW diffraction that follows from Eq. (1). The transmission function is plotted in Fig. 2 for the values of $U = 1$ and $Q = 1$. It is known that the FWHM of the transmission function for conventional Bragg diffraction equals to $H_{FWHM} = 1.6$. The solution for ML pulses gives the 1.5 times greater FWHM value and side lobes are completely smoothed. It is easy to show, that the maxima of oscillating function $T(H)|_{Q=1}$ follow the $1/H^2$ law at $H \gg 1$. The normalized transmission function for wave packets is approximately equal to the envelope of these maxima.

**DISCUSSION**

In the analysis above, we neglected GDD of the crystal, therefore it is necessary to specify the conditions under which the results are valid. It is known that the constraint for using first-order approximation of dispersion is $|\gamma|\Delta \omega^2 L \ll 1$, where $\Delta \omega$ is the optical bandwidth [6]. Calculation for $\lambda = 800$ nm in the paratellurite crystal with $L = 60$ mm gives the condition for pulse duration $\tau \gg 240$ fs. The group delay between the eigenwaves in a crystal is $L/|v|$, that equals to 33 ps for the same crystal parameters. Thus we can say that the analysis in this paper is valid for optical pulses...
with several picosecond duration that corresponds to the bandwidth of few Angstrom. This bandwidth is not wider than the typical passband of collinear acousto-optical filters [12], therefore pulsed light can be efficiently diffracted by single-frequency ultrasound. For femtosecond laser pulses the Eq. (4) must be solved with GDD terms. Moreover, the bandwidth of sub-100-fs is as wide as tens of nanometers that is much greater than the single-frequency passband of acousto-optical devices, hence chirped ultrasound is necessary for broadband light-sound interaction.

SUMMARY

We derived and modified coupled-wave equations describing diffraction of ultrashort optical pulses. A unified theory considers anisotropic Bragg diffraction and group delay between the eigenwaves in the crystal. The results of numerical simulation demonstrate essential distinctions between the conventional and modified theories. It makes evident that precise characterization of acousto-optical diffraction of femtosecond pulses requires solving a PDE problem of Bragg coupling in birefringent media for wave packets.

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